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THE DYNAMIC-BIAS IN RADIATION INTERROGATION OF TWO-PHASE FLOW

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Abstract—An analysis is presented concerning the dynamic-bias which appears in radiation interrogation of fluctuating two-phase flow. It is shown that in certain cases it is possible to obtain closed form expressions for the dynamic-bias. Based on these analytical and calculational results, conditions are enumerated which must be emphasized if the dynamic-bias is to be minimized.

INTRODUCTION

THE USE of various radiations as an interrogation tool in two phase flow studies has in recent years become firmly accepted [1–4]. Concurrent analytical and empirical investigations have been pursued to examine the uncertainty in such measurements attributable to instrumentation characteristics and effect of steady-state flow orientations [4–7]. Additionally, it has recently been shown that in such experiments the dynamic characteristics of the voids can lead to a most significant bias in the experimentally measured void fraction [8].

The identification of a bias attributable to flow voiding characteristics, herein to be designated as the dynamic-bias, now requires that the various contributing system parameters be identified. In addition, it is necessary to determine the relative effect of various time-dependent void fluctuations. An understanding of these factors would, first, clarify the mathematical–physical description of a radiation beam in its passage through a dynamic medium, second, provide guidance in the specification and design of experimental systems and, third, provide an indication of the relative severity of different flow conditions with respect to the dynamic bias.

Here, we summarize the mathematical description of radiation transmission through a two-phase flow system. Subsequently, we describe the derivation of some dynamic-bias representations appropriate to specific voided flow conditions. Finally, we enumerate several conditions which must be emphasized in the general application of radiation transmission in voided flow if the dynamic-bias is to be minimized.

DESCRIBING EQUATIONS

We consider a voided channel for which the void fraction at any time along its traverse may be designated by $\alpha(t)$. The mean void fraction during the time interval τ is defined by

$$\langle\alpha\rangle = \frac{1}{\tau}\int_0^\tau \alpha(t)\,\mathrm{d}t. \tag{1}$$

In the Appendix we derive the general expression for the transmission of a beam of radiation through a voided channel. The expression for the radiation transmittance follows directly and is given by

$$T = \frac{\mathrm{e}^{-\lambda}}{\tau}\int_0^\tau \mathrm{e}^{\lambda\alpha(t)}\,\mathrm{d}t, \tag{2}$$

where $\lambda = \mu_0 x_0$ is the channel thickness in units of mean-free-path with μ_0 as the linear attenuation parameter and x_0 the distance between the containment walls.

We will find it convenient to introduce a reference void, α_r, and a reference transmittance, T_r. The reference void describes time independent voiding and hence is equal to the average:

$$\langle\alpha\rangle_r = \frac{1}{\tau}\int_0^\tau \alpha_r \, dt, \\ = \alpha_r. \tag{3}$$

The reference transmittance, T_r, is defined in terms of this reference void, α_r, and is given by

$$T_r = \frac{e^{-\lambda}}{\tau}\int_0^\tau e^{\lambda\alpha_r}\, dt, \\ = e^{-\lambda(1-\alpha_r)}. \tag{4}$$

DYNAMIC-BIAS REPRESENTATION

The dynamic-bias representation which we seek is to express the deviation from the mean void, $\langle\alpha\rangle$, when the steady-state assumption has been made to identify the reference void, α_r. We therefore define this dynamic-bias by the difference between the reference void and the mean void

$$\Delta\alpha = \alpha_r - \langle\alpha\rangle \tag{5}$$

where $\langle\alpha\rangle$ is given in terms of $\alpha(t)$ by equation (1). An expression for the reference void, α_r, in terms of $\alpha(t)$ is found from the following considerations. Recognizing that a given transmittance, T, can be associated with any number of different void variations, $\alpha(t)$, we may therefore equate equation (4) with equation (2)

$$e^{-\lambda(1-\alpha_r)} = \frac{e^{-\lambda}}{\tau}\int_0^\tau e^{\lambda\alpha(t)}\, dt, \tag{6}$$

and solve for α_r to yield

$$\alpha_r = \frac{1}{\lambda}\ln\left\{\frac{1}{\tau}\int_0^\tau e^{\lambda\alpha(t)}\, dt\right\}. \tag{7}$$

Substituting this expression together with equation (1) into equation (5) yields the dynamic-bias as a functional of the void variation, $\alpha(t)$:

$$\Delta\alpha = \frac{1}{\lambda}\ln\left\{\frac{1}{\tau}\int_0^\tau e^{\lambda\alpha(t)}\, dt\right\} - \frac{1}{\tau}\int_0^\tau \alpha(t)\, dt. \tag{8}$$

By direct substitution, we can show that $\Delta\alpha = 0$ if $\alpha(t)$ is a constant; hence, no bias is encountered if the steady-state assumption, equation (4), is applicable. For the case of void build-up during time τ, for example, the result is significantly different as can be shown. Thus, in this case we write

$$\alpha(t) = \beta t, \qquad 0 \leqslant \beta \leqslant \frac{1}{\tau}, \tag{9}$$

and substitute it into equation (8) to obtain

$$\Delta\alpha = \frac{1}{\lambda}\ln\left\{\frac{1}{\tau}\int_0^\tau e^{\lambda\beta t}\, dt\right\} - \frac{1}{\tau}\int_0^\tau \alpha(t)\, dt, \\ = \frac{1}{\lambda}\ln\left\{\frac{\sin h(\lambda\langle\alpha\rangle)}{\lambda\langle\alpha\rangle}\right\}. \tag{10}$$

Here, for algebraic simplicity, we have substituted for β using the definition for $\langle\alpha\rangle$; thus, we must observe the constraint

$$0 \leqslant \langle\alpha\rangle = \frac{\beta\tau}{2} \leqslant \frac{1}{2}. \tag{11}$$

The non-linearity of equation (10) does not render the dependence of $\Delta\alpha$ on the several system parameters very obvious. However, if we expand equation (10) and retain only the leading terms we get the following approximation for the dynamic-bias:

$$\Delta\alpha \simeq \frac{\lambda\langle\alpha\rangle^2}{6} = \frac{\lambda\beta^2\tau^2}{24}, \qquad \lambda\langle\alpha\rangle \leqslant \sqrt{6}. \tag{12}$$

Here we now note the importance of, first, an experimental system parameter as represented by $\lambda = \mu_0 x_0$, second, the experimental procedures as represented by τ, and third, the fluidic conditions as represented by β. Minimization of the dynamic-error requires that the various parameters be chosen in accordance with equation (12). The sensitivity of $\Delta\alpha$ to changes in μ_0, x_0, β and τ can be established by the usual differential procedures; also, a fractional dynamic-basis representation follows directly from equation (12).

The dynamic-bias representation for the case of void depletion follows in a similar manner. For this case we write

$$\alpha(t) = 1 - \beta t, \qquad 0 \leqslant \beta \leqslant \frac{1}{\tau}, \tag{13}$$

and, following substitution into equation (12), we can show that

$$\Delta\alpha = \frac{1}{\lambda}\ln\left\{\frac{\sinh[\lambda(\langle\alpha\rangle - 1)]}{\lambda(\langle\alpha\rangle - 1)}\right\}, \tag{14}$$

where we require

$$\frac{1}{2} \leqslant \langle\alpha\rangle = 1 - \frac{\beta\tau}{2} \leqslant 1. \tag{15}$$

The conclusions from this result can be shown to be identical to those applicable to the void build-up, equation (10).

In the above analysis we derived dynamic-bias representations for transient conditions during time τ. A contrasting but frequently encountered problem is that of steady-state fluctuations about a mean. Such a case arises in bubbly flow which we may describe by the following. We divide the time interval τ into N equal interval of width ω and specify $\alpha(t)$ by

$$\alpha(t) = \begin{cases} \langle\alpha\rangle - \varepsilon, & t_{n-1} \leqslant t < t_{n-1} + \dfrac{\omega}{2}, \\ \langle\alpha\rangle + \varepsilon, & t_{n-1} + \dfrac{\omega}{2} \leqslant t < t_n, \end{cases} \tag{16}$$

for $n = 1, 2, 3, \ldots, N$. Here, the amplitude ε must be chosen to insure that $0 \leqslant \alpha(t) \leqslant 1$.

The dynamic-bias representation, equation (8), for this square-wave void fluctuation is now given by

$$\Delta\alpha = \frac{1}{\lambda}\ln\left\{\frac{1}{\tau}\sum_{n=1}^{N}\left[\int_{t_n-\omega/2}^{t_n} e^{\lambda(\langle\alpha\rangle+\varepsilon)}\,dt + \int_{t_n}^{t_n+\omega/2} e^{\lambda(\langle\alpha\rangle-\varepsilon)}\,dt\right]\right\} - \langle\alpha\rangle,$$

$$= \frac{1}{\lambda}\ln\left\{e^{\lambda\langle\alpha\rangle}\left[\frac{e^{\lambda\varepsilon} + e^{-\lambda\varepsilon}}{2}\right]\right\} - \langle\alpha\rangle,$$

$$= \frac{1}{\lambda}\ln\{\cosh(\lambda\varepsilon)\}. \tag{17}$$

It is significant to find that the periodicity of the fluction has no effect on the dynamic-bias. The medium thickness expressed in units of mean-free-path and the amplitude are, in this case, the important parameters.

A principle relation of interest becomes again clear if we expand equation (17) and retain only the dominant terms:

$$\Delta\alpha \simeq \frac{\lambda\varepsilon^2}{2}, \qquad \lambda\varepsilon \leqslant \sqrt{2}. \tag{18}$$

The square of the amplitude, ε^2, and the thickness of the medium in units of mean-free-path, λ, are thus the determining factors which lead to the dynamic-bias.

For a distinct class of void variation to be examined, we consider exponential void build-up during time τ. The corresponding expression for $\alpha(t)$ which we choose to use is given by

$$\alpha(t) = 1 - e^{-\omega t}. \tag{19}$$

Here, the parameter ω depends upon the mean void; using equation (1) it is readily shown that ω must satisfy the transcendental equation

$$\langle\alpha\rangle = \frac{e^{\omega\tau} + \omega\tau - 1}{\omega\tau}. \tag{20}$$

Due to the algebraic intractability of the error representation for this case it is necessary to evaluate the dynamic-bias by direct integration of equation (8) subject to the condition imposed by equation (20). The result for $\lambda = 1{\cdot}0$, $2{\cdot}0$ and

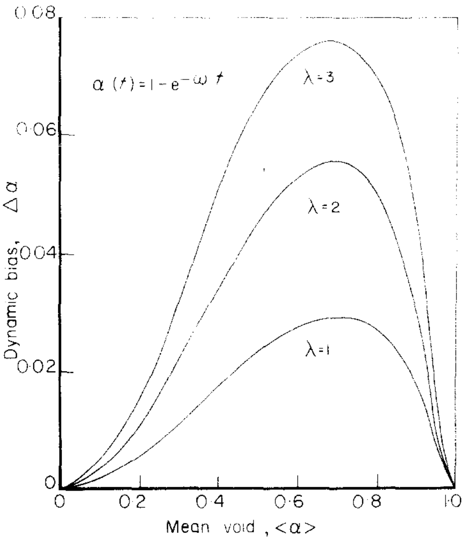

FIG. 1. Dynamic-bias for the case of exponential void build-up defined by $\alpha(t) = 1 - e^{-\omega t}$.

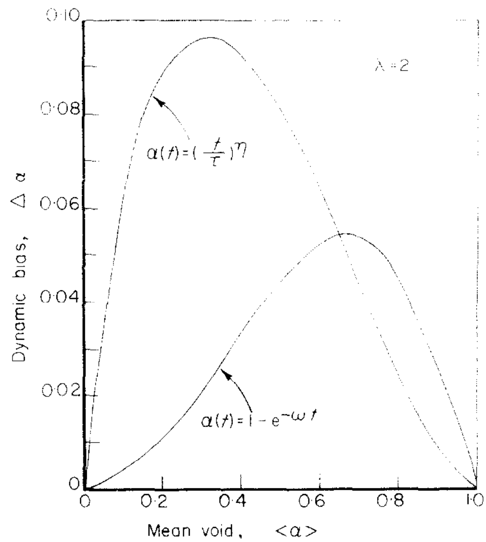

FIG. 2. Effect of different void build-up conditions on the dynamic-bias for the case of $\lambda = 2{\cdot}0$.

3·0 as a function of $\langle\alpha\rangle$ are shown in Fig. 1. Here note, first, the increasing dynamic-bias with increasing λ and, second, a decreasing dynamic-bias for the two extreme cases of zero voiding and full voiding

It is informative to compare the dynamic-bias attributable to the exponential void build-up with a general power law representation of voiding which leads to full voiding at time τ. For this case we write

$$\alpha(t) = \left(\frac{t}{\tau}\right)^{\eta}. \tag{21}$$

where η can be shown to be related to the mean void by

$$\eta = \frac{1}{\langle\alpha\rangle} - 1. \tag{22}$$

We have evaluated the dynamic-bias, equation (8) using equation (21) for the void representation. The results for $\lambda = 2$ are shown in Fig. 2 together with the dynamic-bias arising from the exponential void build-up, equation (19). The noteworthy feature here, is the significant shift in the peak of the maximum dynamic-bias. Thus, the maximum possible dynamic-bias is strongly dependent upon the time dependence of voiding during time τ.

DISCUSSION OF RESULTS

It is informative to discuss the role of the several fluidic conditions as they relate to the fractional dynamic-bias, $\Delta\alpha/\langle\alpha\rangle$. In Fig. 3 we illustrate the property that for the linear void build-up and linear void depletion as well as for the exponential void build-up, most precise measurement can be made at conditions close to either zero or full mean-void; the largest fractional dynamic-bias occurs for $\langle\alpha\rangle \simeq 0{\cdot}5$.

The results from a similar analysis of the fluctuating void (square-wave) variation is significantly different. For the case of constant deviation from the mean void, the fractional dynamic-bias increases significantly with decreasing mean void fraction, Fig. 4; if on the other hand, the deviation ε in a linear function of the mean void fraction then the fractional dynamic-bias increases with $\langle\alpha\rangle$.

Similarly, contrasting results emerge from a

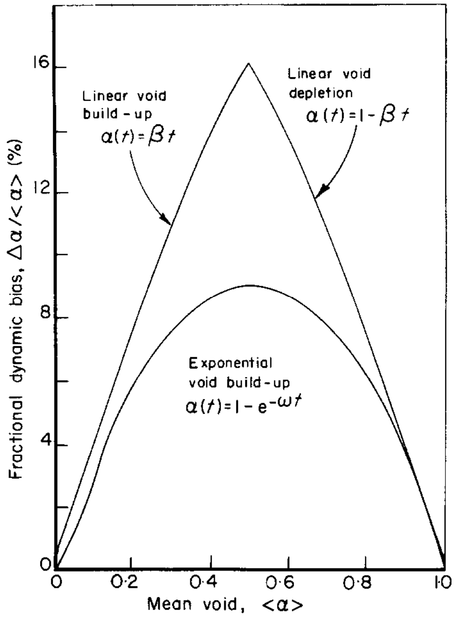

FIG. 3. Fractional dynamic-bias for the case of linear and exponential void fraction variations all for $\lambda = 2{\cdot}0$.

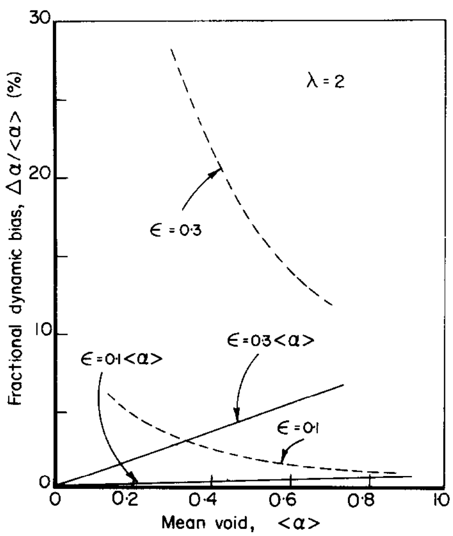

FIG. 4. Fractional dynamic-bias for the case of steady-state (square-wave) fluctuation with different amplitudes.

comparison of the two void build-up representations. For the exponential case, equation (19), the fractional bias is zero in the limits of $\langle\alpha\rangle = 0$ and $\langle\alpha\rangle = 1$ but reaches a maximum in between; its maximum value is 4·9 per cent for $\lambda = 1$ and 9·2 per cent for $\lambda = 2$ with both of these extrema close to $\langle\alpha\rangle \simeq 0{\cdot}52$. However, for the power law void build-up, equation (21), the fractional bias increases monotonically with decreasing mean void, $\langle\alpha\rangle$; its maximum exceeds 30 per cent for $\lambda = 1$ and approaches 82 per cent for $\lambda = 2$ in the limit as $\langle\alpha\rangle$ becomes vanishingly small. These results forcefully emphasize the importance of void variations on the uncertainty in a measured void fraction.

These results serve well to indicate the importance of fluidic conditions and other systems parameters in the determination of the void fractions by radiation interrogation. Several criteria can be cited to provide some guidance for such measurements.

1. The thickness of the fluid, measured in units of mean-free-path, is an important contributor to the dynamic-bias; this contribution can be minimized by using thinner test sections and/or using radiation and media with a low linear attenuation parameter.

2. Transient void variations lead to a lesser dynamic-bias in the limits of zero and full mean voids; this variation, however, is not symmetric about $\langle\alpha\rangle = 0{\cdot}5$. The fractional dynamic-bias may, in some cases, attain a maximum and in other cases increase with decreasing mean void $\langle\alpha\rangle$.

3. Fluctuating voids contribute to an increasing fractional dynamic-bias with decreasing mean void if the deviation is independent of the mean void and to an increasing fractional dynamic-bias with mean void if the deviation is a function of $\langle\alpha\rangle$.

In addition to these criteria, it appears that a recently explored radiation gating procedure [9] might prove particularly useful in such void fraction measurements which are characterized by strong fluctuations. As shown [10], the appropriate transformations from the transmitted signal to the void fraction distribution can provide useful additional information.

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APPENDIX

We consider a narrow, collimated, monoenergetic and time-independent beam of penetrating radiation directed perpendicularly toward a planar channel of liquid containing vertically upwardmoving voids as shown in Fig. 5. The void fraction at time t along the path of radiation is defined by

$$\alpha(t) = \frac{\sum_i \Delta x_i(t)}{x_0}. \tag{A.1}$$

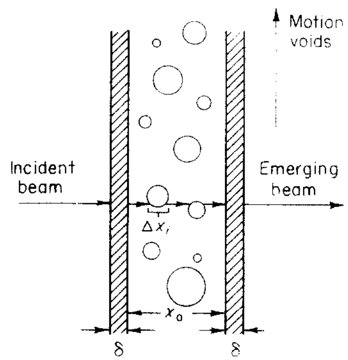

FIG. 5. Schematic representation of voided liquid flow channel.

Recognizing that the total path length through the liquid medium may be written as

$$x_0[1 - \alpha(t)] = x_0 - \sum_i \Delta x_i(t), \tag{A.2}$$

permits relating the incident radiation beam to the emerging radiation beam by the expression

$$I(x_0 + 2\delta) = I(0)\,e^{-\mu_p 2\delta}\,e^{-\mu_0 x_0[1-\alpha(t)]}. \tag{A.3}$$

In this representation we have defined μ_p and μ_0 as the linear attenuation parameters for the channel containment walls and the liquid medium, respectively. It has been assumed that radiation attenuation in the voids can be neglected; it is a straightforward extension to include attenuation in the void, if warranted, by inclusion of the appropriate exponential factor in equation (A.3). Similarly, refractive effects of the two media on the probing radiation have not been included

The detector response without the liquid medium in the channel is given by

$$\begin{aligned} R_0 &= \int_0^\tau \xi I(0)\,e^{-\mu_p 2\delta}\,dt, \\ &= \xi I(0)\,e^{-\mu_p 2\delta}\,\tau, \end{aligned} \tag{A.4}$$

where ξ is the detector efficiency. Similarly, with the voided liquid in the channel, we write

$$\begin{aligned} R_1 &= \int_0^\tau \xi I(0)\,e^{-\mu_p 2\delta}\,e^{-\mu_0 x_0[1-\alpha(t)]}\,dt. \\ &= \xi I(0)\,e^{-\mu_p 2\delta}\,e^{-\mu_0 x_0}\int_0^\tau e^{\mu_0 x_0 \alpha(t)}\,dt. \end{aligned} \tag{A.5}$$

The radiation transmittance, defined by the transmission fraction R_1/R_0, is thus a functional of $\alpha(t)$:

$$T[\alpha(t)] = \frac{e^{-\mu_0 x_0}}{\tau}\int_0^\tau e^{\mu_0 x_0 \alpha(t)}\,dt. \tag{A.6}$$

Note that this analysis does not incorporate spatial effects due to beam-area or variations of α with x.

DEVIATION DANS L'EXPLORATION PAR RAYONNEMENT D'UN ECOULEMENT BIPHASIQUE

Résumé—On présente une analyse de la déviation qui apparaît dans l'exploration par rayonnement d'un écoulement biphasique fluctuant. On montre qu'il est possible dans certains cas d'obtenir des expressions analytiques de la déviation. Basées sur ces résultats théoriques, on donne des conditions exploitables si la déviation est minimisée.

DIE DYNAMISCHE VERZERRUNG BEI STRAHLUNGSRECHNUNGEN IN ZWEI-PHASENSTRÖMUNG

Zusammenfassung—Eine Analyse der dynamischen Verzerrung bei Strahlungsrechnungen in fluktuierender Zwei-Phasenströmung wird angegeben. Es wird gezeigt, dass es in bestimmten Fällen möglich ist, geschlossen darstellbare Ausdrücke für die dynamische Verzerrung zu erhalten. Auf Grund dieser analytischen und rechnerischen Ergebnisse werden die Bedingungen aufgezählt, unter denen die dynamische Verzerrung minimalisiert werden kann.

ДИНАМИЧЕСКОЕ СМЕЩЕНИЕ ПРИ ВОЗДЕЙСТВИИ ИЗЛУЧЕНИЯ НА ДВУХФАЗНОЕ ТЕЧЕНИЕ

Аннотация—Дается анализ динамического смещения, появляющегося при воздействии излучения на пульсирующее двухфазное течение. Показано, что в определенных случаях для динамического смещения можно получить выражения в замкнутом виде. На основе результатов анализа и расчета перечисляются условия, на которые следует обратить внимание, чтобы свести до минимума динамическое смещение.